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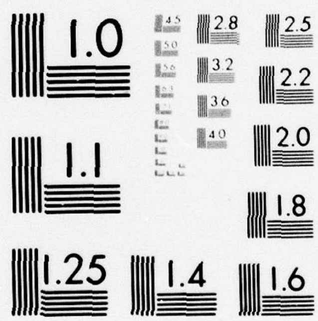

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NPRDC TR 80-12 February 1980

RECRUIT INPUT OPTIMIZATION (RIO) MODEL: FORMULATION AND DEVELOPMENT

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
14 NPRDC-TR-80-12 ✓

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 RECRUIT INPUT OPTIMIZATION (RIO) MODEL: FORMULATION AND DEVELOPMENT.

5. TYPE OF REPORT & PERIOD COVERED
9 Final Report
Jan 1977-Sep 1978.

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10 Yu-Song Yen

8. CONTRACT OR GRANT NUMBER(s)
12 36

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Navy Personnel Research and Development Center
San Diego, California 92152

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
62763N
ZF55-521-010-03.11

11. CONTROLLING OFFICE NAME AND ADDRESS
Navy Personnel Research and Development Center
San Diego, California 92152

12. REPORT DATE
11 February 1980

13. NUMBER OF PAGES
36

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
16 F55521

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
17 ZF55521010

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Linear programming
Goal programming
Optimization
Manpower model
Recruitment
Training management

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A cross-sectional manpower flow model was formulated for the Navy Enlisted Personnel System. The model describes the flow of recruits through the three nonrated pay grades (E-1--E-3) and the third class petty officer pay grade (E-4) into the higher petty officer pay grades (E-5--E-9). The purpose is to assure sufficient input to the petty officer force structure so as to minimize future personnel shortages and surpluses. The report describes the problem of projecting the number of recruits subject to requirement constraints on petty officer force structure and careerists, and the restrictions on training capacity.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

390 772

FOREWORD

This research and development was conducted under Task Area ZF55-521-010 (Management Decision Models) and the sponsorship of the Deputy Chief of Naval Operations (OP-01). The objective of this task area is to develop techniques to improve the Navy's managerial decision-making capabilities. Work Unit 03.11 within this Task Area concerns the flow of recruits through the three nonrated pay grades (E-1--E-3) and the third class petty officer pay grade (E-4) into the higher petty officer pay grades (E-5--E-9). The objective is to assure sufficient input to the petty officer force structure so as to minimize future personnel shortages and surpluses.

Acknowledgments are due to Mr. Joe Silverman, who provided information on the workings of the lower three pay grades, and to Dr. Calvin B. Lee, who provided helpful comments.

DONALD F. PARKER
Commanding Officer

SUMMARY

Problem

To meet the changing demand for petty officers, now and in the future, careful planning of recruit input to the enlisted personnel system is necessary. The problem is complicated by limitations in "A" School training plant capacity, requirements for petty officers, and other constraints on the number of recruits that can be input to the system in any given time period.

Objective

The purpose of this report is to describe a quantitative model for planning recruit input in an optimal manner. This model, which analyzes recruit input from the flow of personnel through the three nonrated pay grades (E-1--E-3) and the third class petty officer pay grade (E-4) to the higher petty officer grades (E-5--E-9), was used experimentally to determine the optimal number of recruits needed to satisfy requirements for future petty officers and careerists. The objective is to minimize future personnel shortages and surpluses.

Approach

The recruit input optimization (RIO) model was formulated within a linear goal programming framework, and was based on the flow of personnel in the enlisted system, the need for petty officers and careerists, and restrictions on training capacity. A penalty function is used to measure the difference between requirements and calculated strength.

Results

1. A linear goal programming model for determining optimal recruit input to the enlisted personnel system to satisfy future petty officer requirements was developed and appears feasible for single ratings or the total Navy. The planning horizon for the model is 10 years.

2. The coefficient matrix and the right-hand side of this linear goal program can be estimated from the historical data. Recruits and future inventories are the decision variables. The linear programming package--IBM 370/165 MPSX--is used to obtain an optimal solution.

3. The approach used in this work cannot satisfy the problem of optimizing recruit input to all ratings simultaneously. Nevertheless, the insights resulting from development of the RIO model were useful in making technical tradeoffs necessary for implementing more restricted versions of the model.

Future Development

A number of necessary or desirable extensions to this model are being considered. These include (1) distributing recruits to "A" School or on-the-job training (OJT), (2) extending the model to include transfers and other interactions among skill ratings, (3) determining the effects of Reserve accessions on the recruitment schedule, (4) determining the effect of budget constraints on implementation of optimal recruitment schedules for each rating, and (5) most important, extending the approach to encompass all ratings.

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INTRODUCTION

Problem and Background

The Navy Enlisted Personnel System is characterized by roughly 450,000 individuals classified in some 100 occupational specialties (ratings), 9 pay grades (PG), and 31 time-in-service (TIS) intervals measured in years.

The pay grade structure is composed of two major components: a petty officer force comprising the top six grades (E-4--E-9) and a "nonrated" or nonpetty officer force consisting of the lower three grades (E-1--E-3). The nonrated force consists largely of apprentices who are undergoing formal or on-the-job training. These individuals usually have no long-term commitment to the Navy. Most important, their advancement is decentralized organizationally and is not directly controlled by personnel planners.

The TIS dimension of the personnel system also can be divided into two components: Personnel with less than 4 years of service are considered noncareer, and those with more than 4 years, career. Generally, personnel with less than 4 years are serving in their first term of enlistment. A substantial portion of noncareer personnel leave the Navy at the end of this period. There is a high probability that those who remain will continue as careerists.

The petty officer force represents the professional enlisted structure necessary to perform and supervise the large number and variety of tasks constituting the Navy's work. To support this force, a number of personnel must be recruited to the Navy each year. The demand for apprentices eligible to become petty officers is generated by the advancement system. Vacancies are created in the petty officer grades through demotions, promotion out of a grade, and losses. These vacancies are accumulated at Pay Grade E-4 and represent the demand for petty officers in a given time period. To satisfy this demand, an adequate number of apprentices at pay grade E-3 must be available at the appropriate time and occupational specialties.

To be eligible for advancement to pay grade E-4, a candidate must serve a minimum time in grade (TIG) at E-3 and time in service (TIS), be recommended by his commanding officer, and (in most cases) have taken and passed the advancement in rating examination. Entry to the petty officer force not only implies requirements for a sufficient level of experience and success in advancement examination, but also denotes some training--either formal classroom training ("A" School) or on-the-job training (OJT). This entry-level training requirement ensures technical competence in a Navy occupational specialty. In the highly technical ratings, such as Electronics Technician, the length and cost of formal training is an important factor in determining both the quantity and quality of recruits. In other ratings, such as Boatswains' Mate, most training is accomplished through OJT.

For a given number of recruits, the portion that flows through "A" School vice OJT is represented by a fraction called the "A" School/OJT split. Some constraint is required to ensure that recruitment levels for any specific time period do not exceed the capacity of the training plant. There is also a need to oscillate the number of personnel that are recruited over time. Because new recruits, for the most part, must go through some schooling at a training facility, highly oscillatory recruitment policies would incur large costs in the repeated opening and closing of facilities, as well as reassignment of staff. Hence, the recruitment levels should be somehow constrained to avoid large fluctuations over time.

There are also some constraints associated with the enlisted system that influence the level of recruitment: (1) budget (this places an upper limit on recruitment), (2) minimum requirements for promotions by TIS and TIG, (3) satisfying "A" School/OJT split, and (4) authorized end strength for each pay grade.

Purpose

The purpose of this report is to describe an experimental model--a recruit input optimization (RIO) model--that could be used to determine the number of recruits needed for input to the enlisted system each time period. The idea is to ensure that an adequate number of apprentices at grade E-3 will be available at the right time to satisfy the need for petty officers--as expressed by required promotions at grade E-4--and for careerists. Simultaneously, the model should (1) control training costs by reducing oscillations in the recruitment schedule, and (2) equalize promotion opportunity among recruit cohorts (i.e., all recruits entering in some specific time period). An experimental model would provide sufficient insight into the recruit optimization problem to develop more operational techniques subsequently.

MODEL FORMULATION

Goal Programming

The Recruit Input Optimization (RIO) model was formulated using goal programming techniques. (Appendix A gives a summary of the structure and objectives of the goal program.) The planning horizon for the model is 10 years. It is assumed the manpower requirements (including petty officers), careerist requirements, and "A" School capacity for each rating are known. The model is designed to obtain the "best" recruitment schedule over the planning horizon for the rating.

A number of independent and dependent variables are defined in the goal programming formulation. In general, independent variables are to be determined by the model via optimal solution, for example, of the number of recruits over the planning horizon. Dependent variables are either estimated from historical data or determined by the users to reflect policy changes within the enlisted system. Examples of dependent variables are continuation, promotion and demotion flows (which are estimated) and penalties, and the proportion of recruits assigned to "A" School training ("A" School/OJT split), which is determined by the users.

Constraints

Constraints consist of both conservation of flow equations and goal equations. Conservation of flow equations describe the movement of personnel from one time period to the next. It systematically relates promotions, demotions, continuations, prior service gains, and recruits.

Goal equations are expressed in terms of inventories, recruits, and goal deviation variables. The objective function is the weighted sum, of all the deviations from the desired goals. Weights are the penalties that users assign to control the underage and overage from the goal.

Conservation of Flow Equations

The following notation is used throughout this report:

T = Planning horizon in quarters.

T_{FY} = Planning periods in FY.

G = Number of pay grades.

K = Maximum time in service.

t = Index for time period $\quad t = 1, 2, ..., T, T+1.$

i,j = Index for pay grades $\quad i = 1, 2, ..., G;$
$\quad j = 1, 2, ..., G.$

k = Index for time in service, $\quad k = 1, 2, ..., K.$
$k=K$ is the sum of all time in service greater than and equal to K.

time period t = Time interval between observation point t and t+1.

$S_{ik}(t)$ = Inventory in pay grade i, at the beginning of time period t, with time in service index k.

$f_{ijk}(t)$ = Number of individuals moved from pay grade i, during time period t, to pay grade j, with time in service k. $f_{ijk}(t)$ consists of only those individuals who remain to the end of the time period; that is, it is a net flow of personnel.

If we assume that promotions are made only to the next higher pay grade, then $f_{i-1,i,k}(t)$ are the promotions from pay grade i-1 into pay grade i with time in service index k, during time period t. If we assume demotions can be made to any lower pay grades, then $f_{i\ell k}(t)$ are the demotions from grade i, during time period t, to pay grade ℓ ($\ell<i$), with time in service index k. To be consistent, $f_{iik}(t)$ is the continuation flow of personnel in pay grade i, during time period t, remaining in pay grade i, with time in service index k. $f_{0,j,k}(t)$ is the number of individuals who enter the system at pay grade j, during time period t, with k periods of prior service experience, particularly when k=0, $f_{0,i,0}(t)$ are the recruits (i.e., those with no prior service experience).

At a given point of time, say t+1, inventory and flow are directly related by the conservation of flow equations described below. The inventory of pay grade i with time in service (k+1) at time (t+1), ($S_{i,k+1}(t+1)$), is the sum of those individuals who are prior service gains to pay grade i with time in service k, ($f_{0,i,k}(t)$), plus:

1. Those who are advanced to pay grade i from pay grade (i-1) with time in service k, ($f_{i-1,i,k}(t)$).

2. Those who continue to stay in pay grade i with time in service k, (f_{iik} t)).

3. Those who are demoted from higher pay grades $\ell(\ell>i)$, with time in service k to pay grade i, ($f_{\ell ik}(t)$), during the time period t:

$$S_{i,k+1}(t+1) = f_{0,i,k}(t) + f_{i-1,i,k}(t) + f_{iik}(t) + \sum_{\ell=i+1}^{G} f_{\ell ik}(t). \quad (1)$$

i = 1, 2, ..., G (pay grades)
k = 1, 2, ..., K-2 (time in service)
t = 1, 2, ..., T (planning periods)

Boundary Conditions:

1. $S_{ik}(1)$ is the beginning inventory

i = 1, 2, ..., G (pay grades)
k = 1, 2, ..., K (time in service)

2. Recruits entering the enlisted system at pay grade i, during time period t, with no prior service experience will be in pay grade i with time in service index 1 at time t+1.

$$S_{i,1}(t+1) = f_{0,i,0}(t). \tag{2}$$

i = 1, 2, 3, ..., G (pay grades)
t = 1, 2, ..., T (planning periods)

3. The inventory of pay grade i with time in service K at time $(t+1),(S_{i,K}(t+1))$, is the sum of two groups of individuals. The first group consists of those individuals who have time in service index K-1 at time t; and the second, of those who have time in service index K at time t. (Time in service index K includes all time in service greater than and equal to K.) In particular, the flow equation takes the form:

$$\begin{aligned} S_{i,K}(t+1) = {} & f_{0,i,K-1}(t) + f_{i-1,i,K-1}(t) + f_{i,i,K-1}(t) \\ & + \sum_{\ell=i+1}^{G} f_{\ell,i,K-1}(t) + f_{0,i,K}(t) + f_{i-1,i,K}(t) \\ & + f_{i,i,K}(t) + \sum_{\ell=i+1}^{G} f_{\ell,i,K}(t). \end{aligned} \tag{3}$$

i = 1, 2, ..., G (pay grades)
t = 1, 2, ..., T (planning periods)

for

i=1, $f_{i-1,i,k}(t) \equiv 0$ (no promotions into pay grade 1)

i=G, $f_{\ell,i,K}(t) \equiv 0$ (no demotions into pay grade G)

k = 1, 2, ..., K (time in service)
ℓ = i+1, ..., G (pay grades)
t = 1, 2, ..., T (planning periods)

The above formulation of the problem has very little structure and, hence, too many variables. Recruitment is the major concern of this model. Thus, to reduce the number of variables, we will assume that the fraction of the inventory in pay grade i with time in service k at time t that flows to pay grade j with time in service k+1 at time t+1 is constant, independent of planning period t and the inventory at time t, $S_{ik}(t)$. Thus:

1. $P_{i-1,k}$ is promotion fraction of the inventory in pay grade i-1 with time in service k, into pay grade i.

$$f_{i-1,i,k}(t) = P_{i-1,k}S_{i-1,k}(t).$$

i = 2, 3, ..., G (pay grades)
(no promotion into pay grade 1)
k = 1, 2, ..., K (time in service)
t = 1, 2, ..., T (planning horizon).

2. C_{ik} is continuation fraction of the inventory in pay grade i with time in service k.

$$f_{iik}(t) = C_{ik}S_{ik}(t).$$

i = 1, 2, ..., G (pay grades)
k = 1, 2, ..., K (time in service)
t = 1, 2, ..., T (planning horizon).

3. D^{ℓ}_{ik} is demotion fraction of the inventory in pay grade ℓ, with time in service k, into pay grade i.

$$f_{\ell,i,k}(t) = D^{\ell}_{ik}S_{\ell k}(t).$$

ℓ = i+1, ..., G (pay grades)
i = 1, 2, ..., G-1 (pay grades)
(no demotions into pay grade G)
k = 1, 2, ..., K (time in service)
t = 1, 2, ..., T (planning periods).

Using promotion, demotion and continuation fractions, one can restate the conservation of flow equations as:

$$S_{i,k+1}(t+1) = f_{0,i,k}(t) + P_{i-1,k}S_{i-1,k}(t) + C_{ik}S_{ik}(t) + \sum_{\ell=i+1}^{G} D^{\ell}_{ik}S_{\ell k}(t). \qquad (1')$$

i = 1, 2, ..., G (pay grades)
k = 1, 2, ..., K-2 (time in service)
t = 1, 2, ..., T (planning periods)

Boundary conditions:

1. $$S_{iK}(t+1) = f_{0,i,K-1}(t) + P_{i-1,K-1}S_{i-1,K-1}(t) + C_{i,K-1}S_{i,K-1}(t) + \sum_{\ell=i+1}^{G} D^{\ell}_{i,K-1}S_{\ell,K-1}(t) + f_{0,i,K}(t) + P_{i-1,K}S_{i-1,K}(t) + C_{i,K}S_{i,K}(t) + \sum_{\ell=i+1}^{G} D^{\ell}_{iK}S_{\ell K}(t). \qquad (4')$$

i = 1, 2, ..., G (pay grades)
t = 1, 2, ..., T (planning periods)

2. $P_{1,k} \equiv 0$ k = 1, 2, ..., K

$D^{\ell}_{G,k} \equiv 0$ k = 1, 2, ..., K

3. $S_{ik}(1) = Inv_{ik}$ (i.e., the beginning inventory)

i = 1, 2, ..., G
k = 1, 2, ..., K

Now, to write the conservation of flow equations in a compact form, the following vectors are defined (all vectors are column vectors, transposes are not indicated):

1. $\underline{S}(t)$ = Inventory vector (which includes recruits at time t)

and

$\underline{S}(t) = (S_{11}(t), S_{12}(t), \ldots, S_{1,K}(t), S_{21}(t), \ldots, S_{G,K}(t))$

(Dimension of $\underline{S}(t) = (G \cdot K) \times 1)$ $\quad t = 1, 2, \ldots, T$

2. $\underline{S}'(t)$ = Inventory vector minus the recruits at time t, and

$\underline{S}'(t) = (S_{12}(t), S_{13}(t), \ldots, S_{1,K}(t), S_{22}(t), \ldots, S_{2,K}(t),$
$S_{32}(t), \ldots, S_{G,K}(t)$

(Dimension of $\underline{S}'(t) = (G \cdot (k-1)) \times 1)$ $\quad t = 1, 2, \ldots, T$

3. $\underline{F}(t)$ = Prior service gains vector during time period t, and

$\underline{F}(t) = (f_{0,1,1}(t), f_{0,1,2}(t), \ldots, f_{0,1,K-1}(t) + f_{0,1,K}(t),$
$f_{0,2,1}(t), \ldots, f_{0,G,K-1}(t) + f_{0,G,K}(t))$

(Dimension of $\underline{F}(t) = ((K-1) \cdot G) \times 1)$. $\quad t = 1, 2, \ldots, T$

Define Q to be the matrix of flow rate coefficients for promotion, continuation, and demotion. It has the following structure:

$$Q = \begin{bmatrix} C_1 & D_1^2 & D_1^3 & D_1^4 & D_1^5 \\ P_1 & C_2 & D_2^3 & D_2^4 & D_2^5 \\ & P_2 & C_3 & D_3^4 & D_3^5 \\ & & P_3 & C_4 & D_4^5 \\ & & & P_4 & C_5 \end{bmatrix}$$

(Dimension of $Q = (K-1) \cdot G) \times (K \cdot G)$

where

1. C_i is the continuation rate matrix for pay grade i, and

$$C_i = \begin{bmatrix} C_{i1} & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ 0 & C_{i2} & 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & C_{i,K-2} & 0 & 0 \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & C_{i,K-1} & C_{i,K} \end{bmatrix}$$

(Dimension of C_i = (K-1) x K) $\quad$ i = 1, 2, ..., G (pay grades).

2. P_i is the promotion rate matrix for pay grade i, and

$$P_i = \begin{bmatrix} P_{i1} & 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ 0 & P_{i2} & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & P_{i,K-2} & 0 & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & P_{i,K-1} & P_{i,K} \end{bmatrix}$$

(Dimension of P_i = (K-1) x K) $\quad$ i = 1, 2, ..., G (pay grades).

3. D_i^ℓ is the demotion rate matrix for pay grade i from pay grade ℓ, and

$$D_i^\ell = \begin{bmatrix} D_{i1}^\ell & 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ 0 & D_{i2}^\ell & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & D_{i,K-2}^\ell & 0 & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & D_{i,K-1}^\ell & D_{i,K}^\ell \end{bmatrix}$$

(Dimension of D_i^ℓ = (K-1) x K) $\quad$ i = 1, 2, ..., G (pay grades)
ℓ = i+1, ..., G (pay grades).

Then, the conservation of flow equations can be written as:

$$\underline{S}'(t+1) = Q\underline{S}(t) + \underline{F}(t) \qquad t = 1, 2, ..., T \text{ (time periods)}$$

$\underline{S}(t)$, inventory vector, is determined by the optimal solution of goal program. Its elements $S_{i,1}(t)$, $i = 1, 2, ..., G$, are the optimal recruitment schedules. $\underline{F}(t)$, prior service gains, and Q, flow rate coefficients, are predetermined constants and are estimated from the historical data.

Goal Equations

The six major requirements and the equations used to attain them are described below:

1. Satisfy need for petty officers, at E-4, defined in terms of required promotions, $\underline{A(t)}$. Vacancies are created in the petty officer pay grades through demotions, losses, and promotions out of a pay grade. These vacancies are accumulated at pay grade E-4 and represent the demand for petty officers in a given time period. Since P_{3k} is the promotion rate for pay grade E-3 with time in service index k, $P_{3k}S_{3k}(t)$ represents the number of advancements made to pay grade E-4 during time period t. $D_1^+(t)$ and $D_1^-(t)$ are goal variables. Let T_A be the minimum TIS requirement for advancement to E-4.

$$A(t) = \sum_{k=T_A}^{K} P_{3k}S_{3k}(t) + D_1^-(t) - D_1^+(t) \quad t = 2, ..., T+1.$$

Penalty $\delta_1^+(t)$ is applied to $D_1^+(t)$ for advancements over the required promotions and penalty $\delta_1^-(t)$ is applied to $D_1^-(t)$ for advancements under the required promotion during time period t.

2. Satisfy requirements for careerists, B(t), for each time period t. The career force is the proportion of the total force with at least T_B time periods of service. (In general, T_B = 4 years = 16 quarters.) $D_2^+(t)$ and $D_2^-(t)$ are goal variables that measure overage and underage from the required career force respectively.

$$B(t) = \sum_{k=T_B}^{K} \sum_{i=1}^{G} S_{ik}(t) + D_2^-(t) - D_2^+(t) \qquad t = 2, ..., T+1.$$

$\delta_2^+(t)$ and $\delta_2^-(t)$ are applied to $D_2^+(t)$ and $D_2^-(t)$, for deviations over and under the goal respectively.

3. Reduce oscillations in recruitment between adjacent time periods. This requirement will reduce basic training costs and help to smooth the subsequent movement of recruit graduates into formal school (entry level) training. It can be accomplished by smoothing the recruit training schedules, and can be represented as upper and lower bounds on recruitment. This requirement is "soft," with increasing penalties if the upper and lower limits are violated.

Let $\alpha(t)$ and $\beta(t)$ be the lower and upper limits on recruitment, respectively. $D_3^+(t)$, $D_3^-(t)$ and $D_3^*(t)$ are goal variables.

$$\alpha(t) = \sum_{i=1}^{G} S_{i1}(t) + D_3^-(t) - D_3^*(t) - D_3^+(t)$$

$$D_3^*(t) \leq \beta(t) - \alpha(t). \qquad t = 2, \ldots, T, T+1 \text{ (planning periods).}$$

Penalty $\delta_3^-(t)$ is applied to $D_3^-(t)$ and penalty $\delta_3^+(t)$ is applied to $D_3^+(t)$ for deviations under and over the acceptable range of recruitment. No penalty is assigned to $D_3^*(t)$, which ensures the recruitment schedules falling between its upper and lower limits.

4. Reduce variance in recruitment from fiscal year to fiscal year. This requirement will help to equalize promotion opportunities between adjacent year cohorts and help to assure adequate numbers of qualified promotion resources each year. It can be accomplished by limiting the proportion between adjacent year cohorts to lie within an interval (g(FY), h(FY)).

$$\sum_{t\epsilon FY+1} \sum_{i=1}^{G} S_{i,1}(t) \leq h(FY) \sum_{t\epsilon FY} \sum_{i=1}^{G} S_{i,1}(t)$$

$$\sum_{t\epsilon FY+1} \sum_{i=1}^{G} S_{i,1}(t) \geq g(FY) \sum_{t\epsilon FY} \sum_{i=1}^{G} S_{i,1}(t)$$

$$FY = 1, 2, \ldots, T_{FY}.$$

5. Satisfy the upper limit on recruitment in a single fiscal year. This requirement is a constraint for the total Navy (ALNAV) but a goal for the rating case. It represents a budgetary limit expressed in terms of the total number of enlisted personnel.

Let C(FY) represent the upper limit on recruitment for each fiscal year, and $D_4^-(FY)$ represent the goal variables for underestimation.

$$C(FY) = \sum_{t\epsilon FY} \sum_{i=1}^{G} S_{i,1}(t) + D_4^-(FY)$$

$$FY = 2, 3, \ldots, T_{FY}.$$

Penalty $\delta_4^-(FY)$ is applied to $D_4^-(FY)$ for deviations under the limit.

6. Satisfy upper and lower bounds on the "A" School training pipeline. For each rating, only certain percentages of recruits will be assigned to "A" School training due to capacity limits or policy, and the remaining recruits will be assigned to on-the-job training (OJT). This requirement is "soft," with increasing penalties if bounds are violated.

Let $\gamma(t)$ be the proportion of recruits assigned to "A" School training, and $\eta(t)$, $\theta(t)$ be the respective lower and upper bounds on "A" School training pipeline. $D_5^+(t)$, $D_5^-(t)$ are goal variables that measure the deviation of the number of recruits assigned to "A" School training from its upper and lower bounds on school capacity respectively. $D_5^*(t)$ is the goal variable that assigns no penalty, and ensures that the number of recruits assigned to "A" School training lie between its upper and lower limits.

$$\eta(t) = \gamma(t) \sum_{i=1}^{G} S_{i1}(t) + D_5^-(t) - D_5^*(t) - D_5^+(t)$$

$$D_5^*(t) \leq \theta(t) - \eta(t) \qquad t = 2, \ldots, T, T+1.$$

Penalty $\delta_5^+(t)$ is assigned to goal variables $D_5^+(t)$ and penalty $\delta_5^+(t)$ is assigned to $D_5^-(t)$ for deviations from the upper and lower limits respectively.

Objective Function

The objective function is the summation (linear combinations) of all the weighted penalties assigned to the corresponding goal variables. That is, the objective function is to minimize Z and

$$Z = \sum_{t=2}^{T+1} \delta_1^-(t)D_1^-(t) + \sum_{t=2}^{T+1} \delta_1^+(t)D_1^+(t) + \sum_{t=2}^{T+1} \delta_2^-(t)D_2^-(t)$$

$$+ \sum_{t=2}^{T+1} \delta_2^+(t)D_2^+(t) + \sum_{t=2}^{T+1} \delta_3^-(t)D_3^-(t) + \sum_{t=2}^{T+1} \delta_3^+(t)D_3^+(t)$$

$$+ \sum_{FY=1}^{T_{FY}} \delta_4^-(FY)D_4(FY) + \sum_{t=2}^{T+1} \delta_5^-(t)D_5^-(t)$$

$$+ \sum_{t=2}^{T+1} \delta_5^+(t)D_5^+(t).$$

With the objective function formulated as described, the program minimizes the total discrepancy between goal achievements and goal targets over all time periods. With different weights on penalties, one can control the priority order of the goals, assuming all other variables with fixed values. (See Appendix B for a summarization of the conservation of flow, goal, and objective function equations.)

The final problem consists of approximately G x (k-1) x T + 6T linear rows and approximately G x K x T + 12T variables, and the density (proportion of nonzero elements) is less than 0.07 percent. It is preferable to use software packages that take advantage of the special structure of the problem.

MODEL APPLICATION

The Recruit Input Optimization (RIO) Model is currently suitable for recruit planning in only those ratings that have a single source population (that is, an apprenticeship group that "feeds" only one rating). It is more useful for testing time-in-service or time-in-grade policies for the Total Navy at the lower three pay grades.

In order to scale down the dimensions of the problem, the time-in-grade (TIG) structure was dropped. Either the TIG or time in-service (TIS) dimension had to be discarded because of size considerations, and TIS data was more readily available. Also, for pay grade E-1, TIS is equivalent to TIG. Finally, questions about the career force can be directly addressed with a TIS structure.

The time periods in the model are quarters (i.e., 3 months). Recruits flow through the lower pay grades at such a fast rate that yearly time periods would not describe the flow very accurately. For example, the mean TIS of E-1 personnel at time of advancement to E-2 is between 4 and 5 months.

The Navy enlisted personnel system is represented by the RIO Model as a three-dimensional structure. The basic processing unit is a 42 x 6 force structure matrix, an array of personnel classified by TIS and pay grade. The first 40 rows correspond to 40 TIS intervals, measured in quarters, the 41st row includes all TIS greater than and equal to 41 quarters, and the 42nd row corresponds to the totals of all 41 rows. The first 5 columns correspond to pay grades E-1, E-2, E-3, E-4, and E-5--E-9, an aggregation of the five highest petty officer grades; and the 6th, to "total" pay grade summed over all pay grades for all TIS intervals. The model is a long-range planning model; hence, the distant horizon of 10 years (i.e., 40 quarters).

Data for generating the inventory are obtained from the Enlisted Master Record (EMR), maintained by the Bureau of Naval Personnel; and data for estimating promotion rates, continuation rates, demotion rates, and prior service gains to the system, from the Survival Tracking File (STF), a collection of information for enlisted persons that is updated quarterly from the EMR.

Petty officer requirements for each rating and pay grade are obtained from the Enlisted Programmed Authorizations Plan published by the Deputy Chief of Naval Operations (OP-01). Careerist requirements, authorized end strength, upper and lower bounds on recruits, and "A" School capacities over the planning horizion are entered by the user. The bounds can be used to reflect budget limitations on recruitment and training policies. Penalties are the coefficients of the objective function. Their values are weights that the user assigns to reflect the relative importance of various goals.

As a numerical example for the model, a 5-year run was made using the Hospital Corpman (HM) rating. Constant prior service gains, promotion rates, and continuance rates are assumed over the 5-year (20 quarters) period. The continuance rates, advancement rates, and prior service gains by pay grade and TIS are provided in Tables 1 through 3; and careerist requirements and required petty officer advancements, in Table 4.

Upper limits on recruitment for each fiscal year are assumed to be 800 over the 5-year planning horizon. Lower and upper bounds on "A" School training pipeline are set at 23.90 and 445.77 respectively. Also, it is assumed that the variance between adjacent year recruitment scheduling should not exceed 10 percent, no demotions occurred, and all HMs are assigned to "A" School training (i.e., set $\gamma(t) \equiv 1$). Under the above assumptions,

20 time period (5 years) projections were obtained using the MPSX/370 linear program package. Table 5 gives the beginning inventory, by pay grade and TIS; and Table 6, a sample output of the optimal solution.

Table 1

Average Continuance Rate (percent)--HM Rating
(FT76, Qtr 2 through FY77, Qtr 4)

TIS IN QTRS	Pay Grade Continuance Rate (%) 1	2	3	4	5-9
1	80.87	94.27	87.17	96.93	75.51
2	15.96	87.97	73.44	97.42	98.21
3	26.66	82.78	96.66	98.96	85.22
4	49.31	40.05	93.73	97.67	99.69
5	57.22	39.73	94.36	97.21	89.29
6	53.84	56.55	94.04	94.50	100.00
7	59.93	63.45	89.49	94.24	100.00
8	82.52	65.71	82.37	91.86	96.98
9	62.69	65.58	70.51	87.71	100.00
10	70.29	70.61	74.15	90.01	97.75
11	91.71	77.29	75.37	92.47	97.56
12	63.06	70.48	72.42	91.47	95.34
13	75.32	73.24	79.03	92.99	97.45
14	67.97	75.61	81.35	91.84	98.04
15	63.86	74.66	77.82	81.29	87.55
16	78.23	54.30	45.56	42.57	57.51
17	64.29	70.39	72.50	79.33	90.51
18	64.29	73.47	75.93	81.14	91.35
19	81.16	67.86	74.80	84.95	94.85
20	81.55	82.83	75.37	81.55	95.67
21	82.87	71.43	77.37	88.19	98.45
22	82.15	85.24	73.76	85.95	96.79
23	78.09	84.78	75.50	85.20	95.82
24	92.01	74.37	74.74	81.34	90.86
25	88.54	68.03	63.31	71.89	92.62
26	85.04	79.45	60.45	70.34	90.27
27	71.10	72.97	77.38	79.75	95.47
28	93.61	95.52	82.38	79.04	95.16
29	93.80	81.62	78.57	83.17	94.98
30	94.08	94.08	80.00	81.92	94.35
31	93.59	93.59	71.43	89.29	93.87
32	93.32	93.32	81.68	84.36	93.68
33	93.17	93.17	74.53	79.25	93.78
34	92.79	92.79	80.53	74.96	92.55
35	91.72	91.72	71.98	65.82	93.51
36	92.51	92.51	69.48	80.95	95.45
37	94.99	94.99	82.81	81.63	95.10
38	94.72	94.72	96.26	76.19	92.53
39	92.32	92.32	70.71	77.86	93.60
40	93.02	93.02	77.50	78.57	92.13
≥41	79.12	70.15	72.62	81.07	93.62
TOTAL	43.74	64.74	82.00	84.63	92.98

Table 2

Average Advancement Rate (percent)--HM Rating
(FY76, Qtr 2 through FY77, Qtr 4)

TIS IN QTRS	Advancement Rate (%) by Pay Grade 1-2	2-3	3-4	4-5
1	9.14	0.22	11.32	0.00
2	71.44	7.60	23.40	1.79
3	49.92	14.70	1.21	0.26
4	26.94	56.84	4.35	0.54
5	24.88	55.32	3.74	0.88
6	18.64	34.65	3.92	3.83
7	20.11	28.03	7.91	4.53
8	20.20	20.28	13.43	4.68
9	2.38	16.20	18.30	5.81
10	0.62	14.59	19.10	5.64
11	0.75	11.76	19.37	4.85
12	3.23	10.11	18.53	3.90
13	0.00	8.27	16.04	4.88
14	6.80	4.12	14.61	5.97
15	0.00	2.52	11.28	5.55
16	0.00	0.41	4.10	3.98
17	0.00	0.00	5.85	7.15
18	0.00	0.00	4.46	6.37
19	0.00	4.76	5.71	6.09
20	0.00	0.00	7.80	6.24
21	0.00	0.00	7.06	4.45
22	0.00	0.00	11.01	5.30
23	0.00	0.00	9.10	4.92
24	0.00	0.00	1.01	6.39
25	0.00	3.57	2.43	7.65
26	0.00	0.00	12.72	10.02
27	0.00	0.00	5.95	11.50
28	0.00	0.00	6.43	12.06
29	0.00	0.00	9.52	10.85
30	0.00	0.00	10.00	7.32
31	0.00	0.00	18.57	8.19
32	0.00	0.00	10.71	6.77
33	0.00	0.00	14.29	11.22
34	0.00	0.00	0.00	15.68
35	0.00	0.00	7.14	19.90
36	0.00	0.00	0.00	14.29
37	0.00	0.00	14.29	9.18
38	0.00	0.00	0.00	4.76
39	0.00	0.00	14.29	14.29
40	0.00	0.00	7.14	7.14
≥41	14.29	0.00	12.38	11.02
TOTAL	42.64	28.38	11.19	4.86

Table 3

Prior Service Gains--HM Rating

TIS (QUARTERS)	Pay Grade E-1	E-2	E-3	E-4	E-5/E-9	TOTAL
1	181	140	69	6	0	396
2	7	222	51	29	0	309
3	1	11	32	0	0	44
4	0	2	15	1	0	18
5	0	1	11	1	0	13
6	0	1	11	2	0	14
7	0	1	8	1	0	10
8	0	0	8	0	0	8
9	0	0	5	1	0	6
10	0	0	3	1	0	4
11	0	0	2	1	0	3
12	0	0	4	1	0	5
13	0	0	3	0	0	3
14	0	0	2	1	0	3
15	0	0	3	2	1	6
16	0	0	4	2	2	8
17	0	0	2	1	2	5
18	0	0	1	1	1	3
19	0	0	1	1	0	2
20	0	0	1	0	0	1
21	0	0	1	0	0	1
22	0	0	1	0	0	1
23	0	0	0	1	1	2
24	0	0	0	1	1	2
25	0	0	0	0	2	2
26	0	0	0	0	1	1
27	0	0	0	0	0	0
28	0	0	1	0	1	2
29	0	0	0	0	0	0
30	0	0	0	0	0	0
31	0	0	0	0	2	2
32	0	0	0	0	0	0
33	0	0	0	0	0	0
34	0	0	0	0	0	0
35	0	0	0	0	0	0
36	0	0	0	0	0	0
37	0	0	0	0	0	0
38	0	0	0	0	2	2
39	0	0	0	0	2	2
40	0	0	0	0	21	21
≥41	0	0	0	0	19	19
TOTAL	189	378	239	54	58	918

Table 4

Careerist/Petty Officer Advancements--HM Rating

Fiscal Year	Requirements	
	Careerist	Petty Officer
1	11,761	2,076
2	11,761	2,226
3	11,761	2,236
4	11,761	2,237
5	11,761	2,226

Table 5

Beginning Inventory--HM Rating

TIS (QUARTERS)	Pay Grade E-1	E-2	E-3	E-4	E-5/E-9
1	92	92	15	1	1
2	207	140	92	18	0
3	69	424	150	71	1
4	26	422	249	58	0
5	17	256	604	60	1
6	8	64	558	115	1
7	4	48	701	142	4
8	10	32	657	240	12
9	4	44	587	539	25
10	2	29	370	669	33
11	5	21	189	553	78
12	4	11	141	426	86
13	5	20	134	591	130
14	3	6	103	472	83
15	2	15	93	495	93
16	1	7	77	389	110
17	2	4	78	291	138
18	2	3	20	176	124
19	2	1	17	124	151
20	1	2	10	96	134
21	0	0	14	92	203
22	1	2	11	51	143
23	0	1	7	44	158
24	0	0	5	47	204
25	0	0	9	61	243
26	1	1	10	69	208
27	0	0	4	27	213
28	0	0	1	18	156
29	0	0	1	22	219
30	0	1	2	6	197
31	0	0	3	6	285
32	0	0	1	14	230
33	0	0	1	12	239
34	0	0	1	9	259
35	0	0	1	6	202
36	0	0	0	5	174
37	0	0	1	1	175
38	0	0	1	1	159
39	0	0	0	5	129
40	0	0	0	0	92
≥41	0	1	2	12	4011
TOTAL	468	1647	4920	6034	9104

Table 6

Enlisted Force Structure for Period 2--HM Rating

TIS (QUARTER)	Pay Grade E-1	E-2	E-3	E-4	E-5/E-9
1	74	124	50	0	0
2	256	235	82	9	0
3	40	493	129	68	0
4	19	396	240	72	0
5	13	178	488	68	0
6	10	107	722	82	0
7	4	39	558	132	2
8	2	32	648	190	7
9	8	23	556	309	14
10	3	29	426	581	18
11	1	20	282	674	43
12	5	16	147	549	47
13	0	8	107	416	71
14	0	15	110	571	45
15	0	5	86	450	110
16	0	11	76	415	109
17	0	0	39	171	80
18	0	0	58	236	147
19	0	0	16	145	125
20	0	0	14	107	151
21	0	0	9	79	134
22	0	0	11	82	200
23	0	0	9	45	142
24	0	0	6	39	155
25	0	0	4	39	189
26	0	0	6	45	232
27	0	0	6	50	196
28	0	0	3	22	207
29	0	0	1	14	151
30	0	0	1	18	211
31	0	0	2	5	187
32	0	0	2	6	270
33	0	0	1	12	217
34	0	0	1	10	226
35	0	0	1	7	241
36	0	0	1	4	191
37	0	0	0	4	167
38	0	0	1	1	167
39	0	0	1	1	149
40	0	0	1	4	123
≥41	0	0	2	10	3861
TOTAL	435	1731	4902	5742	8584

FUTURE DEVELOPMENTS

Although a 5-year test run was successful, the following deficiencies to the model and its data base were identified:

1. The need for including season variations on continuation, advancement, and demotion rates.

2. Estimating some portion of prior service gains as a proportion of previous periods' losses.

3. Developing methods to set up the penalty coefficients for the objective function.

4. Modifying the solution technique to reduce the number of equations and variables of the goal program.

Extensions to this model might include: (1) distributing recruits to "A" School or on-the-job training (OJT), (2) extending the model to include transfers and other interactions between skill ratings; (3) determining the effects of satisfying a portion of recruitment needs through reserve accessions, (4) determining the effect of budgetary constraints on implementation of optimal recruitment schedules for each rating, and (5) extending the model to encompass the optimization of all ratings simultaneously.

Basically, the RIO model served as an experimental tool for evaluating different approaches to the problem of recruit optimization. In addition, the model has since been adapted to operate as an "annualized" version for Total Navy applications. In this form, it has been implemented for use in Navy manpower programming and is known as the Optimal Accession Requirements (OAR) model.

APPENDIX A

SYNOPSIS OF GOAL PROGRAMMING

SYNOPSIS OF GOAL PROGRAMMING

This report applies goal programming, an adaptation of linear programming, to the problems of manpower and personnel planning. In linear programming, the planning problem is represented by a number of constraints and by an objective function to be optimized (cost minimization or profit maximization). The levels of the various activities to be scheduled are represented as variables whose values are to be determined in accordance with the optimization criterion and the conditions imposed by the constraints. In the case of goal programming, a number of goals are admitted and these goals do not have to be mutually compatible. Each goal is represented as an equality constraint, with the addition of two goal variables that represent any under- or overachievement of the goal target.

Let

n = number of activities,

a_{rj} = per unit contribution of the jth activity towards the achievement of goal r, and

G_r = goal target for the rth goal.

The rth goal can be formulated as

$$G_r = \sum_{j=1}^{n} a_{rj}x_j + u_r - v_r$$

where

u_r = under-achievement of the rth goal (underage)

v_r = over-achievement of the rth goal (overage)

$u_r, v_r \geq 0$

If we give the same weight to under- and overachievement and equal weight to each goal, the objective function is then

Minimize $\Sigma(u_r + v_r)$.

The form of the objective function and the way in which u_r and v_r appear in the goal formulations ensure that u_r and v_r cannot both be nonzero for the same value r. Constraints can be included in a goal program. Thus, the goal program formulation consists of a number of constraints representing conditions that must be met, and a number of goals or targets representing the various goals of the manager, with the objective function formulated as described above.

The program minimizes the total discrepancy between goal achievements and goal targets. Alternatively, under- and overachievement can be given different penalty weights and different weightings may be assigned to the various goals. Mathematically, the changes required by a goal program to the usual linear program formulation are

relatively minor. Nevertheless, the use of a goal program structure affects the way the planning problem is represented and the interpretation of results. This applies particularly to the sensitivity analysis available with a linear program. In a goal program, sensitivity measures such as shadow values (duals) and unit cost/profit sensitivities can still be interpreted mathematically in terms of goal satisfaction, although they may no longer have the same economic and practical interpretation. Consequently, there is a case for a flexible mixture of the two techniques. A goal program can be used to generate a number of appropriate solutions according to the relative weights given to the various goals. The program can then be temporarily frozen into a linear program structure at the levels of goal achievement indicated by the goal program to apply the linear program sensitivity analysis with respect to satisfying individual goals in isolation.

APPENDIX B

SUMMARIZATION OF RIO GOAL PROGRAM

SUMMARIZATION OF RIO GOAL PROGRAM

$$\min Z = \sum_{t=2}^{T+1} \delta_1^-(t)D_1^-(t) + \sum_{t=2}^{T+1} \delta_1^+(t)D_1^+(t) + \sum_{t=2}^{T+1} \delta_2^+(t)D_2^+(t)$$

$$+ \sum_{t=2}^{T+1} \delta_2^-(t)D_2^-(t) + \sum_{t=2}^{T+1} \delta_3^-(t)D_3^-(t) + \sum_{t=2}^{T+1} \delta_3^+(t)D_3^+(t)$$

$$+ \sum_{FY=1}^{T_{FY}} \delta_4^-(FY)D_4^-(FY) + \sum_{t=2}^{T+1} \delta_5^-(t)D_5^-(t) + \sum_{t=2}^{T+1} \delta_5^+(t)D_5^+(t) \quad .$$

Subject to:

(1) Conservation of flow equations:

$$f_{0,i,k}(t) = S_{i,k+1}(t+1) - P_{i-1,k}S_{i-1,k}(t) - C_{ik}S_{ik}(t)$$

$$- \sum_{\ell=i+1}^{G} D_{i,k}^{\ell} S_{\ell k}(t)$$

$i = 1, 2, \ldots, G$

$k = 1, 2, \ldots, K-2$

$t = 1, 2, \ldots, T$

(2) Boundary conditions:

$$f_{0,i,K-1}(t) + f_{0,i,K}(t) = S_{iK}(t+1) - P_{i-1,K-1}S_{i-1,K-1}(t)$$

$$- C_{i,K-1}S_{i,K-1}(t) - \sum_{\ell=i+1}^{G} D^{\ell}_{i,K-1}S_{\ell,K-1}(t)$$

$$- P_{i-1,K}S_{i-1,K}(t) - C_{i,K}S_{i,K}(t)$$

$$- \sum_{\ell=i+1}^{G} D^{\ell}_{i,K}S_{\ell,K}(t) \quad .$$

$i = 1, 2, \ldots, G$

$t = 1, 2, \ldots, T$

$S_{i,k}(1) = Inv_{i,k}$ (given beginning inventory)

$i = 1, 2, \ldots, G$

$k = 1, 2, \ldots, K$

(3) Satisfying petty officers force constraints (over all time periods):

$$A(t) = \sum_{k=T_A}^{K} P_{3k}S_{3k}(t) + D_1^-(t) - D_1^+(t) \quad .$$

$t = 2, \ldots, T+1$

(4) Satisfying career force constraints (over all time periods)

$$B(t) = \sum_{k=T_B}^{K} \sum_{i=1}^{G} S_{ik}(t) + D_2^-(t) - D_2^+(t) \quad .$$

$$t = 2, \ldots, T+1$$

(5) Reducing training costs constraints by reducing oscillations in recruitment schedule (over all time periods):

$$\alpha(t) = \sum_{i=1}^{G} S_{i,1}(t) + D_3^-(t) - D_3^*(t) - D_3^+(t)$$

$$D_3^*(t) \leq \beta(t) - \alpha(t). \qquad t = 2, \ldots, T+1$$

(6) Equalizing promotion opportunity between cohorts by reducing the variations in recruitment between adjacent year cohorts (over all fiscal years):

$$\sum_{t \epsilon FY+1} \sum_{i=1}^{G} S_{i,1}(t) \leq h(FY) \sum_{t \epsilon FY} \sum_{i=1}^{G} S_{i,1}(t) \quad .$$

$$\sum_{t \epsilon FY+1} \sum_{i=1}^{G} S_{i,1}(t) \geq g(FY) \sum_{t \epsilon FY} \sum_{i=1}^{G} S_{i,1}(t)$$

$$FY = 1, \ldots, T_{FY}$$

(7) Budget constraints placing an upper limit on recruitment (over all fiscal years):

$$C(FY) = \sum_{t \epsilon FY} \sum_{i=1}^{G} S_{i,1}(t) + D_4^-(FY) \quad .$$

$$FY = 1, \ldots, T_{FY}$$

(8) **Satisfying A-school/OJT split (over all time periods):**

$$\eta(t) = \gamma(t) \sum_{i=1}^{G} S_{i,1}(t) + D_5^-(t) - D_5^*(t) - D_5^+(t) \quad .$$

$D_5^*(t) \leq \theta(t) - \eta(t)$ $\quad\quad$ $t = 2, \ldots, T+1$

(9) **Non-negativity constraints:**

$S_{i,k}(t) \geq 0$ $\quad\quad$ $i = 1, 2, \ldots, G$

$k = 1, 2, \ldots, K$

$t = 2, \ldots, T+1$

$D_j^+(t) \geq 0$ $\quad\quad$ $j = 1, 2, 3, 5$

$t = 2, 3, \ldots, T+1$

$D_j^-(t) \geq 0$ $\quad\quad$ $j = 1, 2, 3, 5$

$t = 2, 3, \ldots, T+1$

$D_4^-(FY) \geq 0$ $\quad\quad$ $FY = 1, 2, \ldots, T_{FY}$

$D_j^*(t) \geq 0$ $\quad\quad$ $j = 3, 5$

$t = 2, 3, \ldots, T+1$

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Principal Deputy Assistant Secretary of the Navy (Manpower and Reserve Affairs)
Chief of Naval Operations (OP-102) (2), (OP-11), (OP-13), (OP-13B), (OP-16), (OP-964D), (OP-987H)
Chief of Naval Research (Code 434), (Code 450) (3), (Code 452), (Code 458) (2)
Chief of Information (OI-2252)
Director of Navy Laboratories
Commandant of the Marine Corps (Code MPI-20)
Chief of Naval Education and Training (N-5)
Commander, Naval Military Personnel Command (NMPC-013C)
Commander, Navy Recruiting Command (Code 20)
Director, Training Analysis and Evaluation Group (TAEG)
Personnel Research Division, Air Force Human Resources Laboratory (AFSC), Brooks Air Force Base
Occupational and Manpower Research Division, Air Force Human Resources Laboratory (AFSC), Brooks Air Force Base
Technical Library, Air Force Human Resources Laboratory (AFSC), Brooks Air Force Base
Personnel Research and Measurement Division, Assistant for Personnel, Plans, and Analysis, Air Force Manpower and Personnel Center, Randolph Air Force Base
Systems Development and Support Division, Directorate of Personnel Data Systems, Air Force Manpower and Personnel Center, Randolph Air Force Base
Headquarters, Department of the Army, Office of the Deputy Chief of Staff or Personnel (Chief, Research Office)
Army Research Institute for the Behavioral and Social Sciences (Reference Service)
Army Research Institute for the Behavioral and Social Sciences Field Unit--USAREUR (Library)
Director for Acquisition Planning, Office of the Assistant Secretary of Defense (Manpower, Reserve Affairs, and Logistics)
Executive Secretary, Defense Advisory Committee on Women in the Services (DACOWITS), OASD(MRA&L) (35)
Commandant, Industrial College of the Armed Forces
Director, Defense Activity for Non-Traditional Education Support
Defense Race Relations Institute
Secretary Treasurer, U.S. Naval Institute
Science and Technology Division, Library of Congress
Commandant, Coast Guard Headquarters (G-P-1/62)
Commanding Officer, U.S. Coast Guard Training Center, Alameda
Commanding Officer, U.S. Coast Guard Institute
Defense Technical Information Center (12)